Int. J. Heat Mass Transfer. Vol. 25, No. 3, pp. 426–428, 1982
Printed in Great Britain

0017-9310/82/030426-03 $03.00/0

HEAT TRANSFER IN VERTICAL GAPS

JEAN N. KOSTER*
Kernforschungszentrum Karlsruhe, Institut für Reaktorbauelemente,
D-7500 Karlsruhe 1, W. Germany

(*Received* 5 *June* 1981 *and in revised form* 13 *August* 1981)

NOMENCLATURE

b,	width of the Hele–Shaw box in x direction;
d,	gap width of the Hele–Shaw box in y direction;
h,	height of the Hele–Shaw box in vertical z direction;
g,	acceleration of gravity;
n,	refractive index;
Pr,	Prandtl number, ν/κ;
Ra,	Rayleigh number, $\beta g \Delta T h^3/\kappa\nu$;
T,	temperature;
ΔT,	temperature difference between lower and upper boundary;
T',	dimensionless temperature, $T(z)/\Delta T$;
x, y, z,	Cartesian coordinates;
Y,	full width of the Hele–Shaw box (walls + gap width d);
z',	dimensionless height, z/h.

Greek symbols

α,	wavenumber;
β,	coefficient of thermal expansion;
κ,	thermal diffusivity;
ν,	kinematic viscosity.

Subscript

c,	critical.

INTRODUCTION

INTERFEROMETRIC studies of hydrodynamic stability in slender vertical gaps (Hele–Shaw boxes) heated from below were performed previously [1, 2] where a detailed description of the box is given. Hele–Shaw boxes are characterized by a large aspect ratio h/d, the third dimension b is in this case $b \gg h$. The horizontal boundaries are made of high conduction copper.

Using real time holographic interferometry the heat transfer at the horizontal boundaries can be determined [3–5]. The advantage of holographic compared to other interferometers is that inhomogeneous optical properties of the transparent test box walls do not influence the interferograms. Thus low quality transparent plastic side walls can be used. A Plexiglas box filled with water approximates the adiabatic boundary condition while a silicone oil filled crystal glass box approximates a perfectly conducting boundary both allowing flow visualization.

Fluid flow in Hele–Shaw boxes is often used to simulate flow through porous media [6]. In this context this investigation is of interest to understand transport phenomena in groundwater and oil flow.

*Present address: Department of Physics, University of Utah, Salt Lake City, UT 84112, U.S.A.

RESULTS AND DISCUSSION

A typical interferogram of convection in a water filled (Pr = 6.3) Plexiglas box h/d = 18.1 with d = 1 mm and 12 mm thick walls is shown in Fig. 1. The integral temperature field of the fluid and the wall is visualized at Rayleigh number $Ra = 2Ra_c$, twice the critical Rayleigh number for the onset of convective flow $Ra_c = 7.5 \times 10^5$. The theoretical origin of this high numerical value of Ra_c, which depends upon h/d and the ratio of the thermal conductivities of the wall material and the fluid, is fully discussed elsewhere [1, 2]. To evaluate the quality of such an interferogram, a plot of numerical calculations [7] of the temperature field of convection in an adiabatic box of same aspect ratio h/d is shown in the same figure. The corresponding streamlines illustrate the connection between the roll cell pattern and the temperature field with an upflow at the center.

Special care must be taken to obtain interferograms of evaluative quality in Plexiglas boxes. In such a convection box the number of fringes is to a large extent due to the side walls [1]. Since the temperature dependence of the reactive index of Plexiglas is of the same order of magnitude as water an integral temperature dependent refractive index dn/dT over the light path Y should be measured before evaluating the interferograms. In this investigation the temperature difference ΔT was measured by thermocouples and the fringe temperature $T(z)$ was calculated via an iteration process determining a mean value of dn/dT along the integration length Y. Achievement of higher accuracy was not attempted. In this way the dimensionless temperature profiles at Fig. 2(a) have been evaluated from the interferogram Fig. 2(b) along the vertical lines A, B, C which are laid through downflow (A), center of roll (B) and upflow (C).

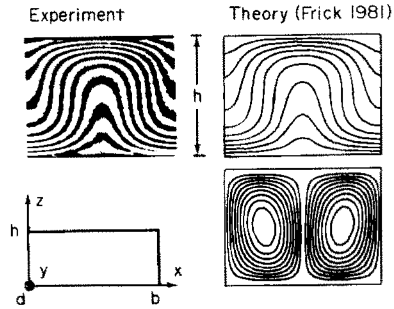

FIG. 1. Comparison of interferogram to calculated isotherms for convection in a Hele–Shaw gap (h/d = 18.1, $Ra = 2Ra_c$ and α = 4.8). Experiment: Side walls—Plexiglas, h/b = 0.042; fluid—Water, Pr = 6.3. Theory: Side walls—Adiabatic, h/b = 0; fluid—$Pr = \infty$.

The fringe number and the vertical location of a fringe is measured from the film negative (interferogram) at an overhead measuring table with 0.1 mm resolution, which is also the overall resolution. The vertical coordinate is made dimensionless with the height, $z' = z/h$, beginning at the lower boundary. The temperature of a fringe at a certain distance z' is determined via the standard procedure [4, 5]. The temperature of each fringe is made dimensionless with the temperature difference applied to the box i.e. $T'(z) = T(z)/\Delta T$. The curve $T'(z)$ is smoothed with a standard polynomial and plotted as a function of z'.

When convective flow sets in, there is at the upper and lower boundaries a fluid layer through which heat is transported only by conduction. In these layers Fourier's law of heat conduction is applicable. To calculate the heat transfer near the horizontal boundaries a parabola is fitted to three points $T'(z')$ close to the horizontal boundaries. At the locations $z' = 0$ and $z' = 1$ a tangent to the parabolas is calculated to obtain the Nusselt number, Nu. As a simple error control Nu was calculated from a linear interpolation of the two closest points to the boundary. The evaluation program was checked with the results of Farhadieh and Tankin [8].

In a Hele–Shaw box with the high conductivity side walls (crystal glass) and silicone oil ($Pr \approx 38$) the temperature dependence of the refractive index dn/dT of the side walls is negligible [1]. The fringe pattern is now solely due to the integration of the light along the gap width d in the box. As d is very small ($d \approx 1$ mm and $d \approx 3$ mm) only a small fringe number is obtained and the temperature difference between fringes is high. Frick [7] has shown that in a box with perfect conduction side walls the temperature profile is non-uniform along d and so the refractive index is dependent upon the integration along d, i.e $n = n(x, y, z)$. Thus the errors in determining the fringe temperature could be reduced to a tolerable level by iterating the temperature of the fringes similar to the procedure mentioned above. The dimensionless temperature profiles in Fig. 3(a) were evaluated from the interferogram in Fig. 3(b).

The Nusselt numbers evaluated from some interferograms are shown in Fig. 4 as a function of Ra. Because of the temperature dependence of the refractive index of Plexiglas and the thickness of the walls some fringes disappear at a higher Ra into the horizontal boundaries. Light rays bent in the entering wall are locally blocked by the copper shims of the boundaries. Thus the interferometrical Nu evaluation in a Plexiglas box is limited to low supercritical Rayleigh numbers depending upon the thickness of the walls. The experiments in a low conductivity ("adiabatic") box with Plexiglas side walls are in good agreement with the numerical results of [7] who investigated an adiabatic box of aspect ratio $h/d = 20$. Similar calculations have been performed by Kvernvold [9] for an adiabatic box of aspect ratio $h/d = \infty$ which conform with the results in the box $h/d = 20$ at low Ra [7].

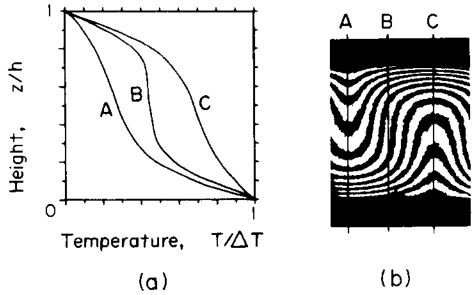

FIG. 2. Dimensionless temperature profiles (a) in a box with low conductivity side walls (Plexiglas) evaluated from the interferogram (b). A, down-flow; B, center of roll; C, upflow. $Ra/Ra_c = 2.17, Pr = 6.3, Nu = 1.9, h/d = 18.1, d = 1.15$ mm.

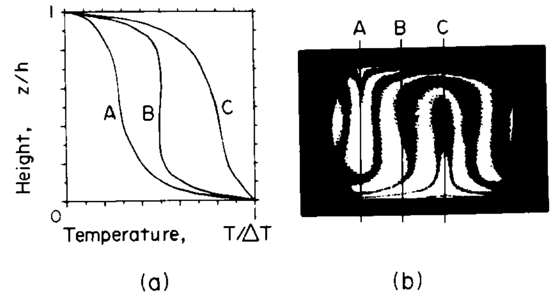

FIG. 3. Dimensionless temperature profiles (a) in a box with high conductivity side walls (crystal glass) evaluated from the interferogram (b). A, down-flow; B, center of roll; C, upflow. $Ra/Ra_c = 3.65, Pr = 38, Nu = 6.0, h/d = 23.35, d = 3.05$ mm.

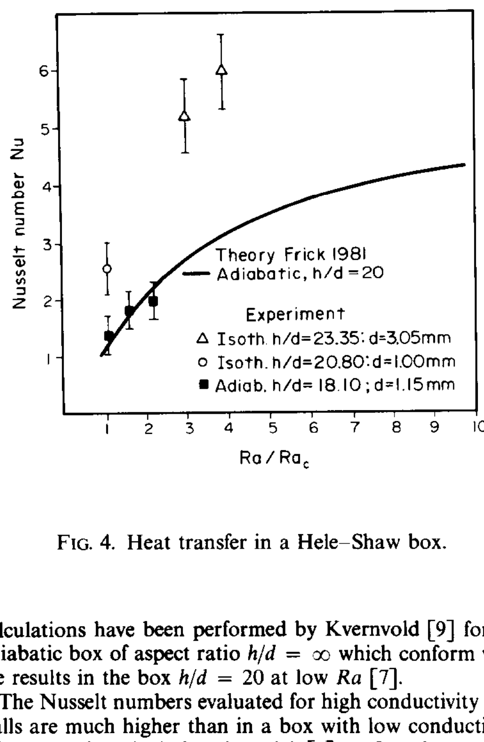

FIG. 4. Heat transfer in a Hele–Shaw box.

The Nusselt numbers evaluated for high conductivity side walls are much higher than in a box with low conductivity side walls. The calculations by Frick [7] confirm the trend to the stronger growth of Nu in perfectly conducting Hele–Shaw boxes. Due to numerical difficulties, however, the heat transfer in Hele–Shaw boxes of aspect ratio $h/d \geq 10$ could only be calculated in the direct neighborhood of the onset of convection.

Acknowledgements—The author wishes to thank J. Abernathey and B. L. Markham for careful reading of the manuscript and for valuable suggestions for improvements. This work has benefited from comments and suggestions of F. Rosenberger. Finally the author would like to express his highest thanks and regards to his dissertation advisor U. Müller.

REFERENCES

1. J. N. Koster, Freie Konvektion in vertikalen Spalten. Dissertation (KfK Report 3066), University of Karlsruhe (1980).

2. J. N. Koster and U. Müller, Free convection in vertical gaps. (to appear in *J. Fluid Mech.*).
3. C. M. Vest, *Holographic Interferometry*. John Wiley, New York (1979).
4. W. Hauf and U. Grigull, Optical methods in heat transfer. In *Advances in Heat Transfer* (edited by J. P. Hartnett and T. F. Irvine, Jr.) Vol. 6, pp. 133–366. Academic Press, New York (1970).
5. R. J. Goldstein, Optical techniques for temperature measurement. In *Measurements in Heat Transfer* (edited by E. R. G. Eckert and R. J. Goldstein) pp. 241–293. McGraw-Hill, New York (1976).
6. J. Bear, *Dynamics of Fluids in Porous Media*. American Elsevier, New York (1972).
7. H. Frick, Zellularkonvektion in Fluidschichten mit zwei festen seitlichen Berandungen. Dissertation (KfK Report 3109), University of Karlsruhe (1981).
8. R. Farhadieh and R. S. Tankin, Interferometric study of two-dimensional Bénard convection cells, *J. Fluid Mech.* **66**, 739–752 (1974).
9. O. Kvernvold, On the stability of non-linear convection in a Hele–Shaw cell, *Int. J. Heat Mass Transfer* **22**, 395–400 (1979).

Int. J. Heat Mass Transfer. Vol. 25, No. 3, pp. 428–430, 1982
Printed in Great Britain

0017 9310/82/030428-03 $03.00/0
Pergamon Press Ltd.

CONVECTION NATURELLE EN MUR PLAN A DES NOMBRES DE PRANDTL ELEVES

R. BEN MAAD
Faculté des Sciences de Tunis, Département de Physique, Belvédère, Tunis

et

A. FORTIER
Laboratoire de Mécanique Expérimentale des Fluides, Bat. 502, Campus d'Orsay, 91400 Orsay, France

(*Reçu le* 15 *mai* 1980 *et sous forme révisée le* 25 *juin* 1981)

NOMENCLATURE

c_p, chaleur spécifique à pression constante [J/kg°C];
g, accélération de la pesanteur [m/s²];
Gr_x^*, $= g\beta x^4\phi/\lambda\nu^2$, nombre de Grashof modifie;
h, $\phi/(T_p - T_\infty)$, coefficient de transfert de chaleur [W/m²°C];
l, longueur de la plaque [m];
Nu_x, hx/λ, nombre de Nusselt;
Nu_x^*, $= Nu_x \cdot (\nu_m/\nu_\infty)^{0.17}$, nombre de Nusselt modifie (zone laminaire turbulente);
Nu_x^{**}, $= Nu_x \cdot (\nu_p/\nu_\infty)^{0.17}$, nombre de Nusselt modifie (zone de transition);
Pr, $= \mu c_p/\lambda$, nombre de Prandtl;
Ra_x^*, $= Gr_x^* \times Pr$, nombre de Rayleigh modifie;
β, coefficient de dilatation du fluide [K⁻¹];
ρ, masse volumique [kg/m³];
μ, viscotié dynamique [kg/ms];
ν, $= \mu/\rho$, viscosité cinématique [m²/s];
ϕ, densité de flux [W/m²].

Indices

DT, les conditions à la limite inférieure du régime transition;
p, les conditions à la température de paroi;
∞, les conditions à la température du fluide loin de la paroi;
m, les conditions à la température du film, $T_m = (T_p + T_\infty)/2$;
x, la valeur locale en abscisse x;
$(\ )_\infty$, propriétés physiques calculées à la température de fluide loin de la paroi;
$(\ )_m$, propriétés physiques calculées à la température du film.

INTRODUCTION

LES ETUDES de convection naturelle sur une plaque plane chauffée à flux où à température constante à des nombres de Prandtl élevés sont rares. A notre connaissance il y a l'étude de Fujii [1, 2] pour les huiles minérales et l'éthylène glycol.

Notre travail [3] sur une plaque plane immergée dans un mélange glicérine–eau à 70% en volume de glycérine vient completer ces études antérieures. Ainsi l'utilisation des fluides visqueux ($Pr > 70$) nous a permis une meilleure connaissance du phénomène de transfert dans les 3 zones (laminaire–transition et turbulence).

PRINCIPE ET ETUDE DU DISPOSITIF EXPERIMENTAL

Nous avons utilisé une installation expérimentale (Fig. 1) qui comprend essentiellement une cuve rectangulaire de 2,40 m de haut, 1,60 m de longueur et 1,10 m de largeur, une plaque plane de 2 m de haut et de 0,90 m de large chauffée par effet Joule, immergée dans le liquide de la cuve et un circuit d'aspiration de la couche limite thermique à la partie supérieure de la plaque. Ce circuit réinjecte après refroidissement le liquide dans la cuve afin de maintenir une température sensiblement constante loin de la plaque et des vitesses aussi faibles que possible. La plaque chauffante est constituée de 3 bandes de clinquant d'acier inoxydable de 0,1 mm d'épaisseur placée côte à côte, tendues verticalement et dans lesquelles circule le courant continu (basse tension) de chauffage. Les températures de la plaque sont mesurées à l'aide de 75 thermocouples chromel–alumel répartis convenablement sur toute la hauteur de la plaque et les températures du fluide loin de la plaque sont mesurées à l'aide de 6 thermocouples eux-même répartis sur la hauteur de la cuve. La puissance électrique dissipée est déterminée à partir de la mesure de la